Module
5
Three-phase AC Circuits

Lesson 19

Three-phase Delta-Connected Balanced Load

In the previous (first) lesson of this module, the two types of connections (star and delta), normally used for the three-phase balanced supply in source side, along with the line and phase voltages, are described. Then, for balanced star-connected load, the phase and line currents, along with the expression for total power, are obtained. In this lesson, the phase and line currents for balanced delta-connected load, along with the expression for total power, will be presented.

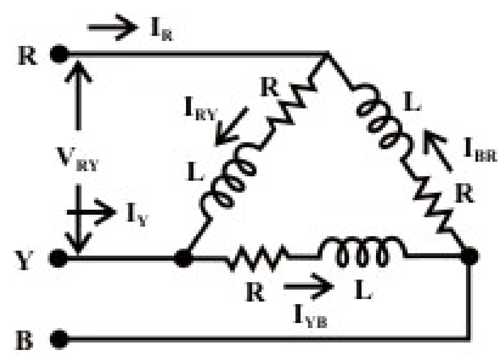
Keywords: line and phase currents, star- and delta-connections, balanced load.

After going through this lesson, the students will be able to answer the following questions:

1. How to calculate the currents (line and phase), for the delta-connected balanced load fed from a three-phase balanced system?

2. Also how to find the total power fed to the above balanced load, for the two types of load connections – star and delta?

Currents for Circuits with Balanced Load (Delta-connected)



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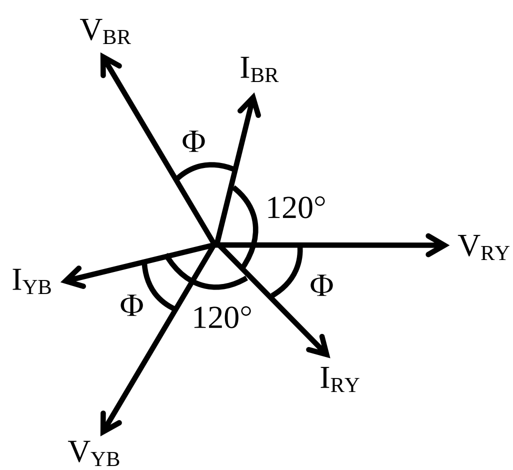


(b)

Fig. 19.1 (a) Balanced delta-connected load fed from a three-phase balanced supply
(b) Phasor diagram

A three-phase delta (Δ)-connected balanced load (Fig. 19.1a) is fed from a balanced three-phase supply. A balanced load means that, the magnitude of the impedance per phase, is same, i.e., $|Z_p| = |Z_{RY}| = |Z_{YB}| = |Z_{BR}|$, and their angle is also same, as $\phi_p = \phi_{RY} = \phi_{YB} = \phi_{BR}$. In other words, if the impedance per phase is given as, $Z_p \angle \phi_p = R_p + j\,X_p$, then $R_p = R_{RY} = R_{YB} = R_{BR}$, and also $X_p = X_{RY} = X_{Yb} = X_{BR}$. The magnitude and phase angle of the impedance per phase are: $Z_p = \sqrt{R_p^2 + X_p^2}$, and $\phi_p = \tan^{-1}\left(X_p / R_p\right)$.In this case, the magnitudes of the phase voltages $|V_p|$ are same, as those of the line voltages $|V_L| = |V_{RY}| = |V_{YB}| = |V_{BR}|$. The phase currents (Fig. 19.1b) are obtained as,

$$I_{RY} \angle -\phi_p = \frac{V_{RY} \angle 0°}{Z_{RY} \angle \phi_p} = \frac{V_{RY}}{Z_{RY}} \angle -\phi_p$$

$$I_{YB} \angle -(120° + \phi_p) = \frac{V_{YN} \angle -120°}{Z_{YB} \angle \phi_p} = \frac{V_{YB}}{Z_{YB}} \angle -(120° + \phi_p)$$

$$I_{BR} \angle (120° - \phi_p) = \frac{V_{BR} \angle +120°}{Z_{BR} \angle \phi_p} = \frac{V_{BR}}{Z_{BR}} \angle (120° - \phi_p)$$

In this case, the phase voltage, V_{RY} is taken as reference. This shows that the phase currents are equal in magnitude, i.e., ($|I_p| = |I_{RY}| = |I_{YB}| = |I_{BR}|$), as the magnitudes of the voltage and load impedance, per phase, are same, with their phase angles displaced from each other in sequence by $120°$. The magnitude of the phase currents, is expressed as $|I_p| = |(V_p / Z_p)|$.

The line currents (Fig. 19.1b) are given as

$$I_R \angle -\theta_R = I_{RY} - I_{BR} = I_p \angle(-\phi_p) - I_p \angle(120° - \phi_p) = \sqrt{3}\, I_p \angle -(30° + \phi_p)$$
$$= I_L \angle -(30° + \phi_p)$$

$$I_Y \angle -\theta_Y = I_{YB} - I_{RY} = I_p \angle -(120° + \phi_p) - I_p \angle(-\phi_p) = \sqrt{3}\, I_p \angle -(150° + \phi_p)$$
$$= I_L \angle -(150° + \phi_p)$$

$$I_B \angle -\theta_B = I_{BR} - I_{YB} = I_p \angle(120° - \phi_p) - I_p \angle -(120° + \phi_p) = \sqrt{3}\, I_p \angle(90° - \phi_p)$$
$$= I_L \angle(90° - \phi_p)$$

The line currents are balanced, as their magnitudes are same and $\sqrt{3}$ times the magnitudes of the phase currents ($|I_L| = \sqrt{3} \cdot |I_p|$), with the phase angles displaced from each other in sequence by $120°$. Also to note that the line current, say I_R, lags the corresponding phase current, I_{RY} by $30°$.

If the phase current, I_{RY} is taken as reference, the phase currents are

$I_{RY} \angle 0° = I_p\,(1.0 + j0.0)$: $I_{YB} \angle -120° = I_p\,(-0.5 - j0.866)$;
$I_{BR} \angle +120° = I_p\,(-0.5 + j0.866)$.

The line currents are obtained as

$$I_R = I_{RY}\angle 0° - I_{BR}\angle +120° = I_p\{(1.0 + j\,0.0) - (-0.5 + j\,0.866)\} = I_p(1.5 - j\,0.866)$$
$$= \sqrt{3}\,I_p\angle -30° = I_L\angle -30°$$
$$I_Y = I_{YB}\angle -120° - I_{RY}\angle 0° = I_p\{(-0.5 - j\,0.866) - (1.0 + j\,0.0)\} = -I_p(1.5 + j\,0.866)$$
$$= \sqrt{3}\,I_p\angle -150° = I_L\angle -150°$$
$$I_B = I_{BR}\angle +120° - I_{YB}\angle -120° = I_p\{(-0.5 + j\,0.866) - (-0.5 - j\,0.866)\}$$
$$= I_p(j1.732) = \sqrt{3}\,I_p\angle +90° = I_L\angle +90°$$

Total Power Consumed in the Circuit (Delta-connected)

In the last lesson (No. 18), the equation for the power consumed in a star-connected balanced circuit fed from a three-phase supply, was presented. The power consumed per phase, for the delta-connected balanced circuit, is given by

$$W_p = V_p \cdot I_p \cdot \cos\phi_p = V_p \cdot I_p \cdot \cos(V_p, I_p)$$

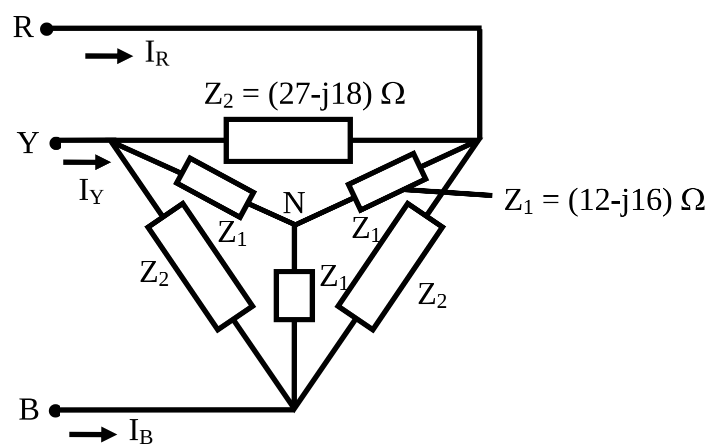
It has been shown earlier that the magnitudes of the phase and line voltages are same, i.e., $|V_p| = |V_L|$. The magnitude of the phase current is $(1/\sqrt{3})$ times the magnitude of the line current, i.e., $|I_p| = (|I_L|/\sqrt{3})$. Substituting the two expressions, the total power consumed is obtained as

$$W = 3 \cdot V_L \cdot (I_L/\sqrt{3}) \cdot \cos\phi_p = \sqrt{3}\,V_L \cdot I_L \cdot \cos\phi_p$$

It may be observed that the phase angle, ϕ_p is the angle between the phase voltage V_p, and the phase current, I_p. Also that the expression for the total power in a three-phase balanced circuit is the same, whatever be the type of connection – star or delta.

Example 19.1

The star-connected load having impedance of $(12 - j16)\ \Omega$ per phase is connected in parallel with the delta-connected load having impedance of $(27 + j18)\ \Omega$ per phase (Fig. 19.2a), with both the loads being balanced, and fed from a three-phase, 230 V, balanced supply, with the phase sequence as R-Y-B. Find the line current, power factor, total power & reactive VA, and also total volt-amperes (VA).



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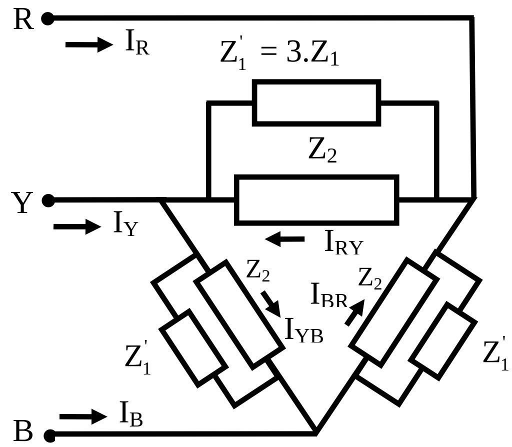

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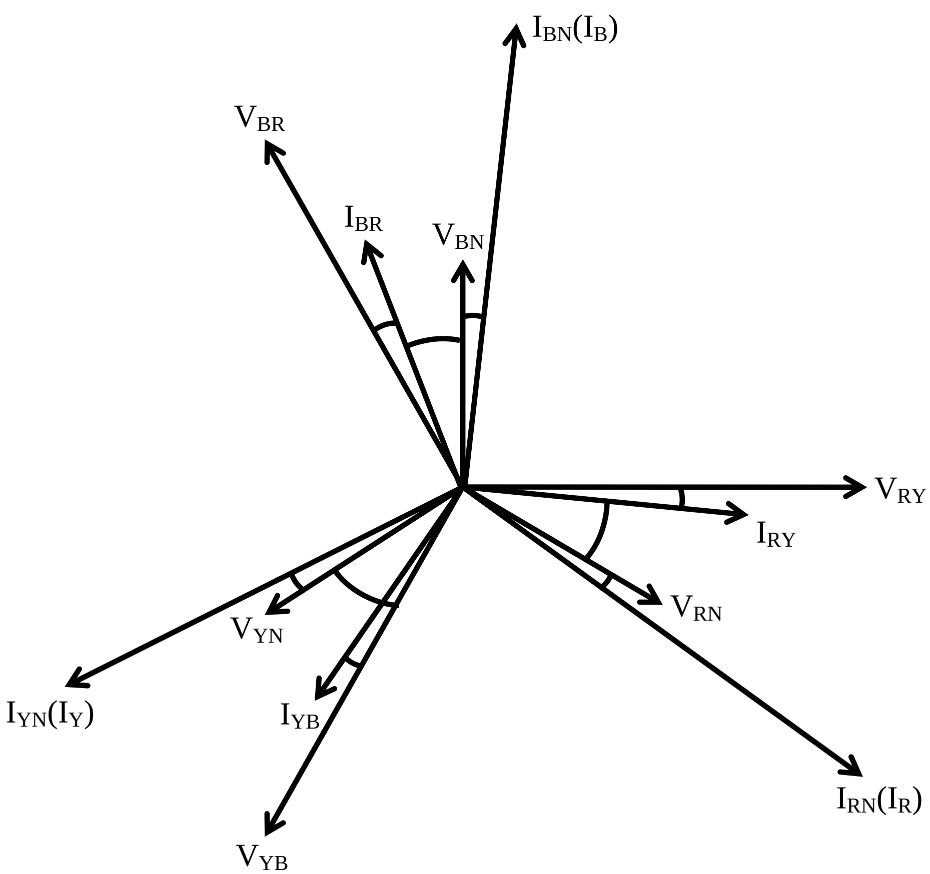


(c)

Fig. 19.2 (a) Circuit diagram (Example 19.1)
(b) Equivalent circuit (delta-connected)
(c) Phasor diagram

Solution

For the balanced star-connected load, the impedance per phase is,

$Z_1 = (12 - j16) = 20.0\ \angle -53.13°\ \Omega$

The above load is converted into its equivalent delta. The impedance per phase is,

$Z_1' = 3 \cdot Z_1 = 3 \times (12 - j16) = (36 - j48) = 60.0 \angle -53.13°\ \Omega$

For the balanced delta-connected load, the impedance per phase is,

$Z_2 = (27 + j18) = 32.45\ \angle +33.69°\ \Omega$

In the equivalent circuit for the load (Fig. 19.2b), the two impedances, Z_1' & Z_2 are in parallel. So, the total admittance per phase is,

$$Y_p = Y_1' + Y_2 = \frac{1}{Z_1'} + \frac{1}{Z_2} = \frac{1}{60.0 \angle -53.13°} + \frac{1}{32.45 \angle +33.69°}$$

$= 0.0167 \angle +53.13° + 0.03082 \angle -33.69°$

$= [(0.01 + j0.01333) + (0.02564 - j0.017094)] = (0.03564 - j0.003761)$

$= 0.03584 \angle -6.024°\ \Omega^{-1}$

The total impedance per phase is,

$Z_p = 1/Y_p = 1/(0.03584 \angle -6.024°) = 27.902\ \angle +6.024° = (27.748 + j\ 2.928)\ \Omega$

The phasor diagram is shown in Fig. 19.2c.

Taking the line voltage, V_{RY} as reference, $V_{RY} = 230\ \angle 0°\ V$

The other two line voltages are,

$V_{YB} = 230\ \angle -120°$; $V_{BR} = 230 \angle +120°$

For the equivalent delta-connected load, the line and phase voltages are same.

So, the phase current, I_{RY} is,

$$I_{RY} = \frac{V_{RY}}{Z_p} = \frac{230.0\ \angle 0°}{27.902\ \angle +6.024°} = 8.243\ \angle -6.024° = (8.198 - j\ 0.8651)\ A$$

The two other phase currents are,

$I_{YB} = 8.243 \angle -126.024°$; $I_{BR} = 8.243 \angle +113.976°$

The magnitude of the line current is $\sqrt{3}$ times the magnitude of the phase current.

So, the line current is $|I_L| = \sqrt{3} \cdot |I_p| = \sqrt{3} \times 8.243 = 14.277\ \ A$

The line current, I_R lags the corresponding phase current, I_{RY} by $30°$.

So, the line current, I_R is $I_R = 14.277\ \angle -36.024°\ \ A$

The other two line currents are,

$I_Y = 14.277 \angle -156.024°$; $I_B = 14.277 \angle +83.976°$

Also, the phase angle of the total impedance is positive.

So, the power factor is $\cos\phi_p = \cos 6.024° = 0.9945\ lag$

The total volt-amperes is $S = 3 \cdot V_p \cdot I_p = 3 \times 230 \times 8.243 = 5.688\ kVA$

The total VA is also obtained as $S = \sqrt{3} \cdot V_L \cdot I_L = \sqrt{3} \times 230 \times 14.277 = 5.688\ kVA$

The total power is $P = 3 \cdot V_p \cdot I_p \cdot \cos\phi_p = 3 \times 230 \times 8.243 \times 0.9945 = 5.657\ kW$

The total reactive volt-amperes is,

$Q = 3 \cdot V_p \cdot I_p \cdot \sin\phi_p = 3 \times 230 \times 8.243 \times \sin 6.024° = 597.5\ VAR$

An alternative method, by converting the delta-connected part into its equivalent star is given, as shown earlier in Ex. 18.1.
For the balanced star-connected load, the impedance per phase is,
$Z_1 = (12 - j16) = 20.0 \angle -53.13° \ \Omega$
For the balanced delta-connected load, the impedance per phase is,
$Z_2 = (27 + j18) = 32.45 \angle +33.69° \ \Omega$
Converting the above load into its equivalent star, the impedance per phase is,
$Z_2' = Z_2 / 3 = (27 + j18)/3 = (9 + j6) = 10.817 \angle +33.69° \ \Omega$
In the equivalent circuit for the load, the two impedances, Z_1 & Z_2' are in parallel.
So, the total admittance per phase is,

$$Y_p = Y_1 + Y_2' = \frac{1}{Z_1} + \frac{1}{Z_2'} = \frac{1}{20.0 \angle -53.13°} + \frac{1}{10.817 \angle +33.69°}$$

$$= 0.05 \angle +53.13° + 0.09245 \angle -33.69° = [(0.03 + j\,0.04) + (0.0769 - j\,0.05128)]$$

$$= (0.1069 - j\,0.01128) = 0.1075 \angle -6.0235° \ \Omega^{-1}$$

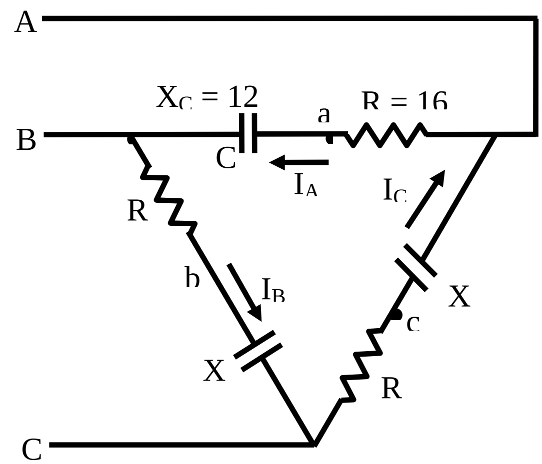
The total impedance per phase is,
$Z_p = 1/Y_p = 1/(0.1075 \angle -6.0235°) = 9.3023 \angle +6.0235° = (9251 + j\,0.976) \ \Omega$
The phasor diagram is shown in Fig. 18.5c. The magnitude of the phase voltage is,
$|V_{RN}| = |V_p| = |V_L| / \sqrt{3} = 230/\sqrt{3} = 132.8 \ V$
The line voltage, V_{RY} is taken as reference as given earlier. The corresponding phase voltage, V_{RN} lags V_{RY} by $30°$. So, the phase voltage, V_{RN} is $V_{RN} = 132.8 \angle -30°$
The phase current, I_{RN} is,

$$I_{RN} = \frac{V_{RN}}{Z_p} = \frac{132.8 \angle -30°}{9.3023 \angle +6.0235°} = 14.276 \angle -36.0235° \ A$$

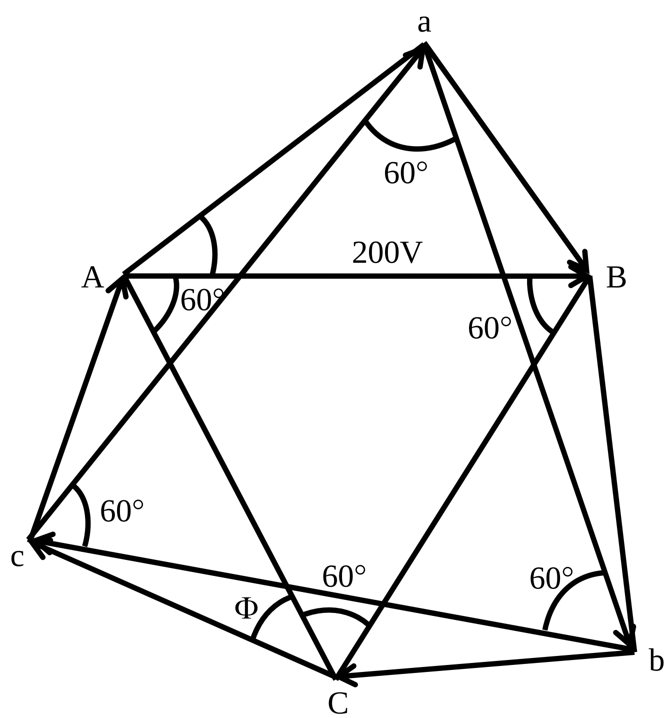
As the total load is taken as star-connected, the line and phase currents are same, in this case. The phase angle of the total impedance is positive, with is value as $\phi = 6.0235°$. The power factor is $\cos\ 6.0235° = 0.9945 \ lag$
The total volt-amperes is $S = 3 \cdot V_p \cdot I_p = 3 \times 132.8 \times 14.276 = 5.688 \ kVA$
The remaining steps are not given, as they are same as shown earlier.

Example 19.2

A balanced delta-connected load with impedance per phase of $(16 - j12)\ \Omega$ shown in Fig. 19.3a, is fed from a three-phase, 200 V balanced supply with phase sequence as A-B-C. Find the voltages, V_{ab}, V_{bc} & V_{ca}, and show that they (voltages) are balanced.



(a)



(b

Fig. 19.3 (a) Circuit diagram (Example 19.2)
(b) Phasor diagram

Solution

$R_p = 16\ \Omega \quad ; \qquad X_{Cp} = 12\ \Omega$

$Z_{AB} = Z_p = R_p - j\,X_{Cp} = 16 - j12 = 20\ \angle -36.87^\circ\ \ \Omega$

For delta-connected load, $|V_L| = |V_p| = 200\ \ V$

Taking the line or phase voltage V_{AB} as reference, the line or phase voltages are,

$V_{AB} = 200\ \angle 0^\circ\,; \qquad V_{BC} = 200\ \angle -120^\circ\,; \qquad V_{CA} = 200\ \angle +120^\circ$

The phasor diagram is shown in Fig. 19.3b. The phase current, I_{AB} is,

$I_{AB} = V_{AB} / Z_p = (200\ \angle 0^\circ)/(20\ \angle -36.87^\circ) = 10.0\ \angle +36.87^\circ = (8.0 + j\,6.0)\ \ A$

The other two phase currents are,

$I_{BC} = 10.0\ \angle -83.13^\circ = (1.196 - j\,9.928)\ \ A$

$I_{CA} = 10.0\ \ \angle +156.87^\circ = (-9.196 + j\,3.928)\ \ A$

The voltage, V_{ab} is,

$V_{ab} = V_{aB} + V_{Bb} = (-j\,X_{Cp}) \cdot I_{AB} + R_p \cdot I_{BC}$

$= (12 \times 10)\angle(36.87^\circ - 90^\circ) + (16 \times 10)\angle -83.13^\circ = 120\angle -53.13^\circ + 160\angle -83.13^\circ$

$= (72.0 - j\,96.0) + (19.14 - j158.85) = (91.14 - j\,254.85) = 270.66\angle -70.32^\circ\ \ V$

Alternatively,

$V_{ab} = (-j12) \times (8.0 + j\,6.0) + 16 \times (1.196 - j\,9.928) = (91.14 - j\,254.85)$

$= 270.66\angle -70.32^\circ\ \ V$

Similarly, the voltage, V_{bc} is,

$V_{bc} = V_{bC} + V_{Cc} = (-j\,X_{Cp}) \cdot I_{BC} + R_p \cdot I_{CA}$

$= (12 \times 10)\angle -(83.13^\circ + 90^\circ) + (16 \times 10)\angle 156.87^\circ = 120\angle -173.13^\circ + 160\angle 156.87^\circ$

$= -(119.14 + j\,14.35) + (-147.14 + j\,62.85) = (-266.28 + j\,48.5)$

$= 270.66\angle +169.68\ \ V$

In the same way, the voltage, V_{ca} is obtained as $V_{ca} = 270.66\angle +49.68^\circ\ \ V$

The steps are not shown here.

The three voltages, as computed, are equal in magnitude, and also at phase difference of 120° with each other in sequence. So, the three voltages can be termed as balanced ones.

A simple example (20.3) of a balanced delta-connected load is given in the following lesson

The phase and line currents for a delta-connected balanced load, fed from a three-phase supply, along with the total power consumed, are discussed in this lesson. Also some worked out problems (examples) are presented. In the next lesson, the measurement of power in three-phase circuits, both balanced and unbalanced, will be described.

Problems

19.1 A balanced load of (9-j6) Ω per phase, connected in delta, is fed from a three phase, 100V supply. Find the line current, power factor, total power, reactive VA and total VA.

19.2 Three star-connected impedances, Z_1 = (8-jb) Ω per phase, are connected in parallel with three delta-connected impedances, Z_2 = (30+j15) Ω per phase, across a three-phase 230V supply. Find the line current, total power factor, total power, reactive VA, and total VA.

List of Figures